Smoothing of line objects with optimization techniques

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1. Introduction

In recent years use of optimization techniques in the field of automated generalization has been proposed by a number of authors (Bobrich 1996, Hójholt 1997). In the main, such techniques are applied to the displacement of line objects through two approaches - energy minimization methods of *snakes* (Burghardt and Meier 1997, Barrault, M., Bader, M. and Weibel, R. 2000) and *beams* (Bader 2001). The methods both resolve proximity conflicts and also minimize shape distortions.

Now, it is proposed to apply snakes to the smoothing of line objects. The advantage of applying one common method for different generalization tasks lies in the easier control of interaction between displacement and smoothing. The paper presents some theoretical background, shows an implementation and discusses some examples of this approach. The automated segmentation of line segments is applied to control the adjustment of parameters.

2. Approach for smoothing of line objects

In this paper the snake approach will be used in the original form, proposed by Kass *et al.* (1987).

$$E(\mathbf{d}) := \int_l \left(\mathbf{E}^{int} + \mathbf{E}^{ext}\right) \cdot ds = \int_l \left(\tfrac{1}{2}(\alpha(s)\cdot\left|\mathbf{d}'(s)\right|^2 + \beta(s)\cdot\left|\mathbf{d}''(s)\right|^2) + \mathbf{E}^{ext}\right)\cdot ds \qquad (1)$$

$$s \mapsto \mathbf{d}(s) = \left(x(s), y(s)\right)^T, \qquad 0 \le s \le l \qquad (2)$$

The differences between displacement and smoothing depend on the definition of vector **d**, which contains only the coordinates of the altered line. For displacement, here the differences between initial line and derived line are used. To find the stable state of the snake the functional $E(\mathbf{d})$ has to be minimized. The variation of $E(\mathbf{d})$ leads to the Eulerian equations, which are solved with discretization in time (Burghardt 2000, Bader 2001). The differences in the final formulas between displacement (3*a,b*) and smoothing (4*a,b*) are straightforward.

$$(\mathbf{A}+\lambda\mathbf{I})\cdot\left(\mathbf{x}^t-\mathbf{x}^0\right) = \lambda\left(\mathbf{x}^{t-1}-\mathbf{x}^0\right) - \mathbf{E}_x^{ext}\left(\mathbf{x}^{t-1},\mathbf{y}^{t-1}\right) \quad (3a) \qquad (\mathbf{A}+\lambda\mathbf{I})\cdot\mathbf{x}^t = \lambda\,\mathbf{x}^{t-1} \quad (4a)$$

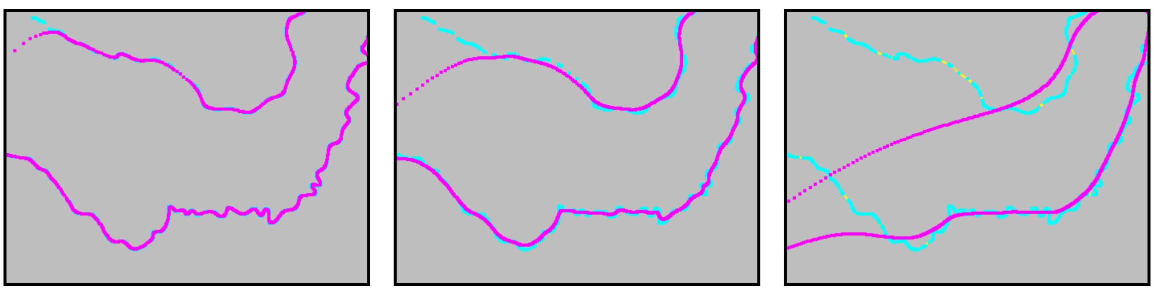
$$(\mathbf{A}+\lambda\mathbf{I})\cdot\left(\mathbf{y}^t-\mathbf{y}^0\right) = \lambda\left(\mathbf{y}^{t-1}-\mathbf{y}^0\right) - \mathbf{E}_y^{ext}\left(\mathbf{x}^{t-1},\mathbf{y}^{t-1}\right) \quad (3b) \qquad (\mathbf{A}+\lambda\mathbf{I})\cdot\mathbf{y}^t = \lambda\,\mathbf{y}^{t-1} \quad (4b)$$

$$\mathbf{A} = \begin{bmatrix} 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & 0 & 0 \\ -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 & 0 \\ \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta & 0 \\ 0 & \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta & \beta \\ 0 & 0 & \beta & -\alpha-4\beta & 2\alpha+6\beta & -\alpha-4\beta \\ 0 & 0 & 0 & \beta & -\alpha-4\beta & 2\alpha+6\beta \end{bmatrix} \quad (5)$$

For smoothing of line objects no external forces are modeled, so the derivatives of external energy ($\mathbf{E}^{ext}$) in x- and y-direction are zero. In contrast to displacement changes in the shape of the line objects are allowed. A simple interpretation for minimizing first and second derivatives of line vector **d** is to minimize distances between points of vector **d** and also minimize the curvature of the spline curve.

3. Technical details of implementation

Depending on the ratio of smoothing and the distance between points, the beginning and the end of the lines are shifted away from their initial position (see Figure 1).



a) $\alpha = 10$ b) $\alpha = 100$ c) $\alpha = 1000$

Figure 1: Defects at the beginning and end of line objects

To overcome this problem there are some workarounds. One possibility is to use first and last point multiply. That results in forcing the smoothed line through the boundary points (see Figure 2).

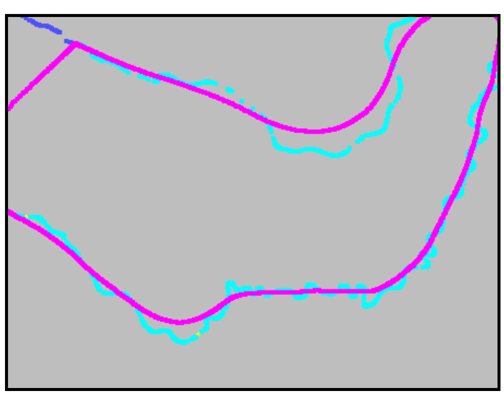
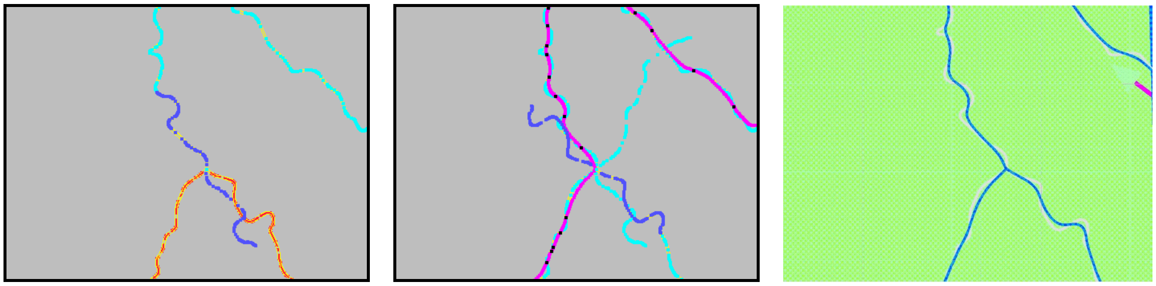


Figure 2: Using boundary points in multiple way ($\alpha = 1000$)

The second option is to extend the original line e.g. with 180° rotation of points at both ends of the lines. So the character of the lines is well represented.



a) $\alpha = 50$ **b) $\alpha = 50$** **c) resulting lines still connected**

Figure 3: Using 180° rotation of boundary points

A important question for any automated solution is the number of points required to bound the solution with respect to the number of points desired. One solution could be found by smoothing without additional boundary points and subsequently counting the distances between the original and smoothed lines. If this distance falls below a threshold value we used the point-id to determine the number of additional boundary values.

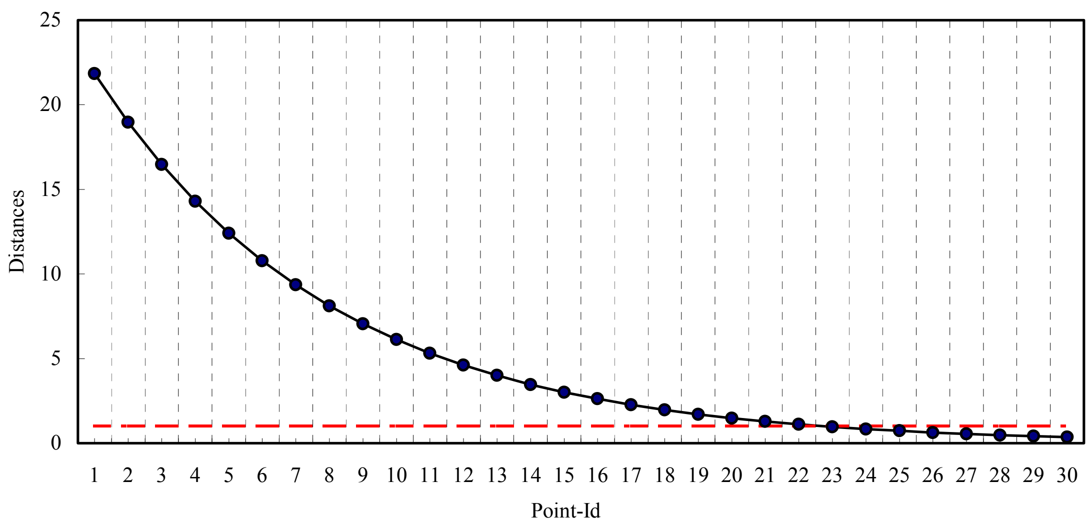


Figure 4: Distances between original and smoothed line

For example in the diagram point-id 24 falls below the threshold value 1.0. This value will be increased by 20%, so we recompute with an additional 5 boundary values.

4. Example

Before smoothing the line objects the segmentation will be done to quantify the parameters α and β of $\mathbf{E}^{int}$. For segmentation a first attempt is executed with default parameters. The points of intersection between the original and smoothed lines correlate with the sinusity of the spline curve. A measure is obtained through counting the intersection points with reference to segment length. In case the distance between two points of intersection is less then a given threshold value the segment is sinuous.

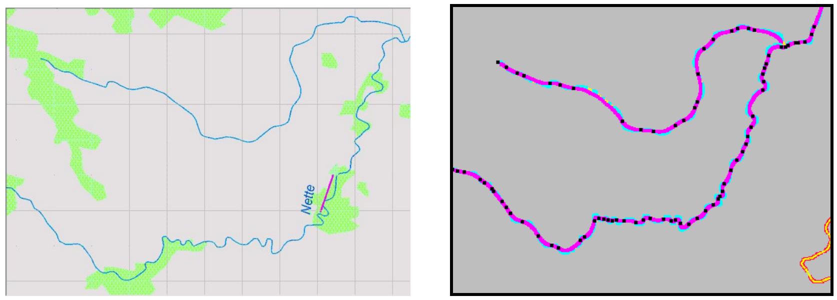


Figure 5: Original and smoothed line with intersection points

The next step is to concatenate segments until a determined minimal length of segments is reached. It is necessary to start the operation with the shortest segments.

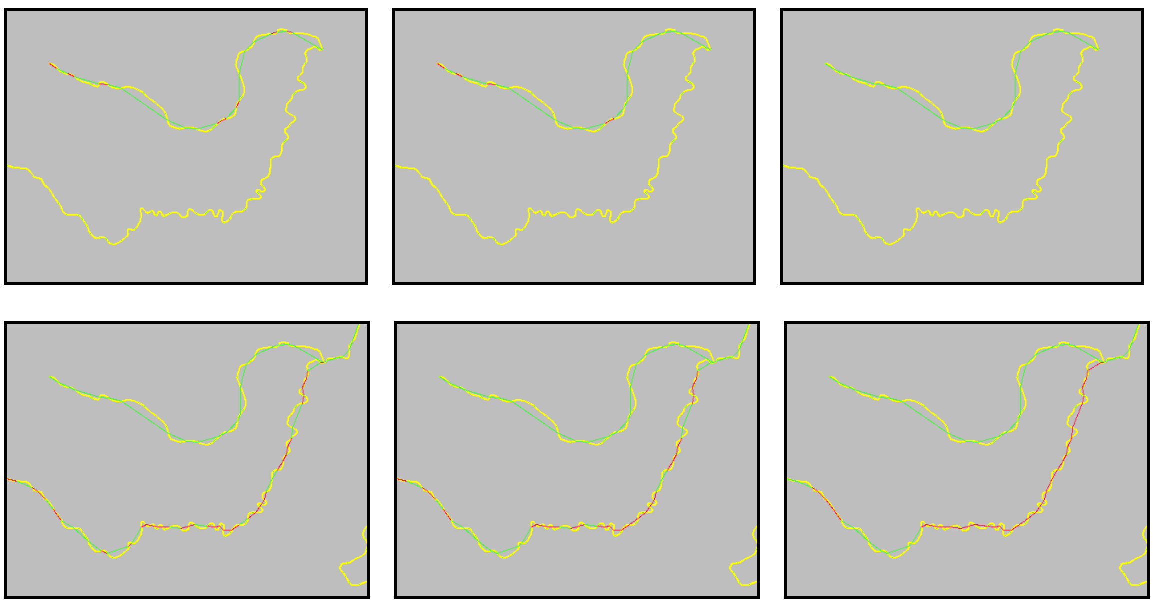
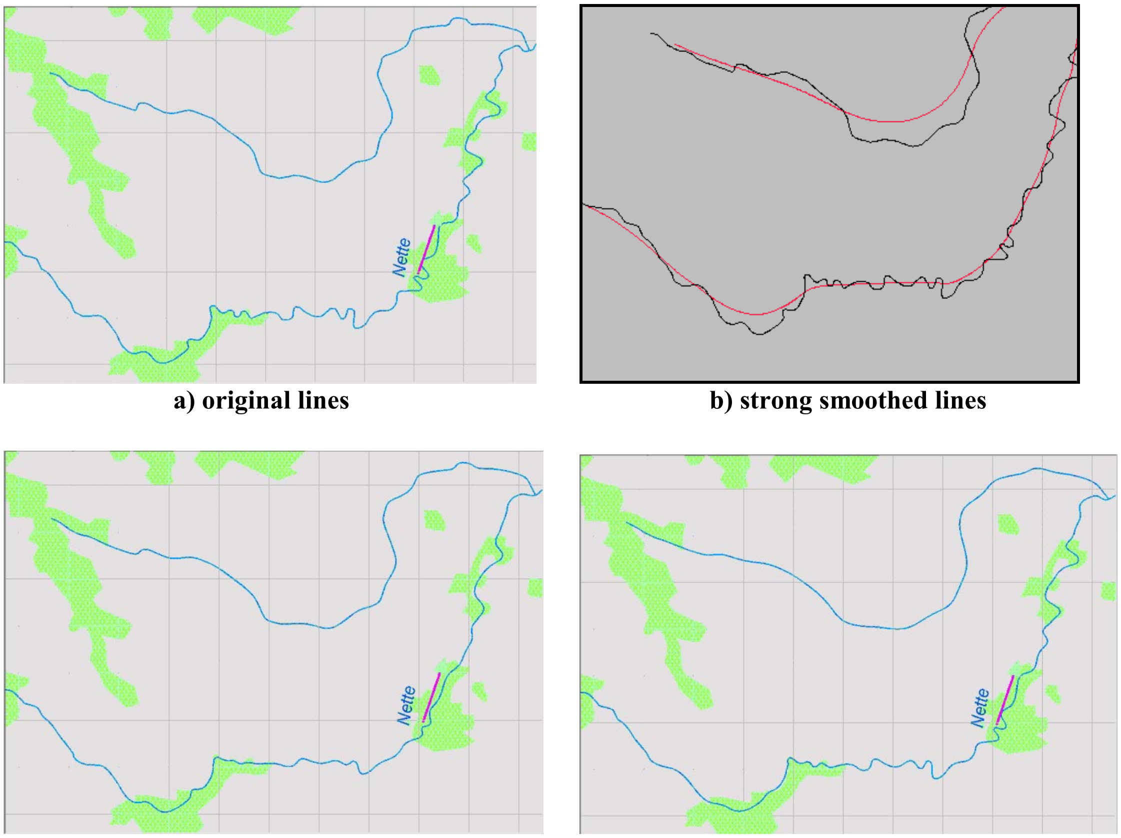


Figure 6: Segmentation of line objects with high (red) and low (green) sinuousity

If there are adjacent segments with different sinuousity (see Figure 6), the longer one determines the value of this attribute. After segmentation the lines are subdivided and smoothed with different parameter values for α and β of the internal energy.

Lines which are sinuous will be smoothed more to eliminate the high frequeny arcs. For the other lines, which are smoothed less, the single curves should be preserved (see Figure 7c). The last figure (7d) shows the reverse result if there is a stronger smoothing of smooth lines. To keep the sinuous parts it is necessary to apply typification rules.



a) original lines **b) strong smoothed lines**

c) stronger smoothing of sinuous lines **d) stronger smoothing of smooth lines**

Figure 7: Results of smoothing with different parameter values α and β

5. Conclusion

This paper presents the applicability of snakes for line smoothing. The main difference of using snakes for displacement or smoothing depends on the definition of internal energy. The advantage of having one common approach for smoothing and displacement is the easier control of interaction between displacement and smoothing. In addition it is faster to apply smoothing and displacement together then one after the other.

The disadvantage of this approach is the selection of values for parameter α and β and resulting deviation from original line. So further work should develop an automatic control of these parameters. A first step is the suggested segmentation to find more sinuous lines. Secondly, it is important to include typification to keep the sinuous parts.

6. Literatur

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